the equation to a cone having (x', y', z') for vertex, and (3) for base.

$$(8)\text{ is } \frac{z'^2}{a^2}x^2+\frac{z'^2}{b^2}y^2+\left(\frac{x'^2}{a^2}+\frac{y'^2}{b^2}-1\right)z^2-\frac{2y'z'}{b^2}yz-\frac{2x'z'}{a^2}xz+2z'z-z'^2=0\ ..(9).$$

The conditions for (9) to be a cone of revolution are $x'=0$,

$$\frac{z'^2}{a^2}-\frac{y'^2}{b^2-a^2}=1\ldots\ldots(10),$$

an hyperbola for the required locus.

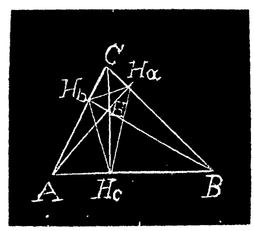
Also solved by *G. B. M. ZERR*.

100. **Proposed by CHARLES CARROLL CROSS, Libertytown, Md.**

O, O_1, O_2, O_3 are the centers of the inscribed and three escribed circles of a triangle ABC. Prove $AO.AO_1.AO_2.AO_3=AB^2.AC^2$.

I. **Solution by the PROPOSER.**

Consider the ex-central triangle ABC as the original triangle. Then $H_aH_bH_c$ is the pedal triangle, and the incenter O becomes the orthocenter H.



Hence we have to prove $AH_c\times BH_c\times CH_c\times HH_c=H_aH_c^2\times H_bH_c^2$.

We readily find by trigonometry that

$$AH_c=\frac{b^2+c^2-a^2}{2c},\quad BH_c=\frac{a^2+c^2-b^2}{2c},$$

$$CH_c=\frac{\sqrt{[4a^2c^2-(a^2+c^2-b^2)^2]}}{2c}=\frac{2\triangle}{c},\quad HH_c=\frac{(b^2+c^2-a^2)(a_2+c^2-b^2)}{8c\triangle},$$

$$H_aH_c=\frac{b(a^2+c^2-b^2)^2}{2ac},\quad H_bH_c=\frac{a(b^2+c^2-a^2)}{2bc}.$$

Substituting in the problem we have

$$\frac{b^2+c^2-a^2}{2c}\times\frac{a^2+c^2-b^2}{2c}\times\frac{2\triangle}{c}\times\frac{(b^2+c^2-a^2)(a^2+c^2-b^2)}{8c\triangle}$$

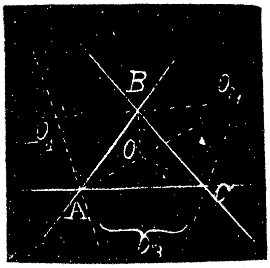
$$=\frac{b^2(a^2+c^2-b^2)^2}{4a^2c^2}\times\frac{a^2(b^2+c^2-a^2)^2}{4b^2c^2}.$$

Since these equations cancel, the proposition is proved.

Mr. Cross should have been credited with solutions of problems 96 and 98 in Geometry, and 100 and 101 in Arithmetic.

II. Solution by **WALTER H. DRANE**, Graduate Student at Harvard University, and **J. SCHEFFER, A. M.**, Hagerstown, Md.

Let ABC be the given triangle, O, O_1, O_2, O_3, the centers of the inscribed and the three escribed circles. Then O_1BO_2, O_1AO_3, O_2CO_3, and AOO_2 are straight lines; also OB, OA, OC are each perpendicular to O_1BO_2, O_1AO_3, and O_2CO_3, respectively. In triangles AOC and BAO_2, $\angle OAC = BAO_2$ and $\angle OCA = \angle BO_2A$ since we have $\angle OCA = 90^\circ - \angle ACO_3 = 90^\circ - \frac{1}{2}(\angle CAB + \angle CBA) = 90^\circ - (\angle OAB + OBA) = 90^\circ - [180^\circ - (\angle O_1AB + \angle O_1BA)] = 90^\circ - \angle O = \angle O_1O_2A$.



$\because$ triangles AOC and ABO_2 are similar, and we have

$$AO : AB :: AC : AO_2 \quad \ldots\ldots\ldots (1).$$

Again in triangles O_1BA and AO_3C, $\angle O = \angle ACO_3$ and $\angle O_3AC = \angle O_1AB$. Hence triangles O_1BA and AO_3C are similar, and we have,

$$AO_1 : AC :: AB : AO_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Multiplying (1) by (2) $AO.AO_1 : AB.BC : AB.AC : AO_2.AO_3$.

$\because AO.AO_1.AO_2.AO_3 = AB^2.AC^2$. Q. E. D.

III. Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Mathematics and Science, Chester High School, Chester, Pa.

In the figure of the last solution, draw the lines OD and O_1D_1 perpendicular to AC. Then $AO = \sqrt{(OD^2 + AD^2)} = \sqrt{(r^2 + r^2\cot^2 \frac{1}{2}A)} = r\operatorname{cosec}\frac{1}{2}A$.

Similarly, $AO_1 = r_1\operatorname{cosec}\frac{1}{2}A$.

$AO_2 = \sqrt{(OO_2^2 - AO^2)} = AO\sqrt{[(OO_2^2/AO^2) - 1]} = AO\cot\frac{1}{2}C$.

Similarly, $AO_3 = AO\cot\frac{1}{2}B$.

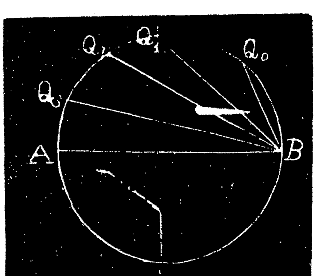
$$\begin{aligned}\therefore AO.AO_1.AO_2.AO_3 &= r^3r_1\operatorname{cosec}^4 \tfrac{1}{2}A \cot\tfrac{1}{2}B\cot\tfrac{1}{2}C \\ &= (s-a)(s-b)(s-c)s\operatorname{cosec}^4 \tfrac{1}{2}A \tan^2 \tfrac{1}{2}A \\ &= s(s-a)(s-b)(s-c)/\sin^2 \tfrac{1}{2}A \cos^2 \tfrac{1}{2}A \\ &= 4s(s-a)(s-b)(s-c)/\sin^2 A = 4S^2/\sin^2 A.\end{aligned}$$

But $\sin A = 2S/bc = 2S/AB.AC$. $\therefore AO.AO_1.AO_2.AO_3 = AB^2.AC^2$.

Also solved by *ELMER SCHUYLER*.

101. Proposed by **E.W. MORRELL, A.M.**, Late Professor of Mathematics, Montpelier Seminary, Montpelier, Vt.

AB is the diameter of a circle and Q_0 any point on the circumference; Q_1, Q_2, $Q_3 \ldots$ are the points of bisection of the arcs AQ_0, AQ_1, $AQ_2 \ldots$ Prove that BQ_1, BQ_2, $BQ_3 \ldots BQ_n = OA^n.(AQ_0/AQ_n)$.



Solution by **G. B. M. ZERR, A. M., Ph. D.**, Professor of Mathematics and Science, Chester High School, Chester, Pa.; **J. SCHEFFER, A.M.**, Hagerstown, Md., and **ELMER SCHUYLER**, High Bridge, N. J.

Let O be the center of the circle.

$\angle ABQ_0 = \theta$.

$\therefore BQ_1 = AB\cos\frac{1}{2}\theta$, $BQ_2 = AB\cos(\theta/2^2)$.

$BQ_3 = AB\cos(\theta/2^3)$, $BQ_n = AB\cos(\theta/2^n)$.